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Coupling Parameters Between a Dielectric Resonator and a Microstripline

P. GUILLON, B. BYZERY, AND M. CHAUBET

Abstract —**The coupling coefficient between a microstripline and the dipolar magnetic ($TE_{0\gamma\delta}$) mode of a cylindrical dielectric resonator is evaluated by using two methods: numerical and analytical.**

Theoretical and experimental external quality factors, as well as the scattering matrix parameters which consider the substrate material, the ground plane, and the distance between the line and the resonator, are presented.

I. INTRODUCTION

INTEREST IN THE utilization of high dielectric constant resonators has been renewed recently because of the availability of low-loss temperature-stable materials [1], [2].

With these new technological advances in the dielectric material, practical applications of dielectric resonators in microwave devices like filters or oscillators were finally deemed feasible [3].

In a previous paper [4], we have studied the coupling between a microstripline and a dielectric resonator. The analysis uses the finite-element method to compute the magnetic field value in the dielectric resonator produced by the current flowing in the microstrip. This method gives good agreement between theoretical and experimental results, but it is not easy to use; so we propose here another similar approach, one also used in [5].

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The aim of this paper is to describe two techniques for the calculation of coupling coefficient parameters which use successively the finite-difference method and an analytical one.

II. GENERAL ANALYSIS

It is well known that to couple a cylindrical dielectric resonator (permittivity ϵ_r, diameter D, height H) acting on its dipolar $TE_{0\gamma\delta}$ mode with a microstripline it is necessary to place the dielectric resonator on the plane of the substrate (permittivity ϵ_s, thickness h_s) as the magnetic lines of the resonator link those of the microstripline (Fig. 1).

A. External Quality Factor Q_e

An equivalent low-frequency network of this system [4] has been analyzed and is shown in Fig. 2. The coupling between the line and the resonator is characterized by a mutual inductance L_m. $L_r, C_r, R_r, L_1, C_1, R_L$ represent, respectively, the equivalent parameters of the resonator and of the line.

The input impedance Z_i calculated in the coupling plane is given by

$$Z_i = \frac{Z(\omega_0)}{1+jX} \tag{1}$$

$$Z(\omega_0) = \omega_0 Q_0 \frac{L_m^2}{L_r} \tag{2}$$

$$X = \frac{2\Delta\omega}{\omega_0} \quad \text{with } \Delta\omega = \omega - \omega_0.$$

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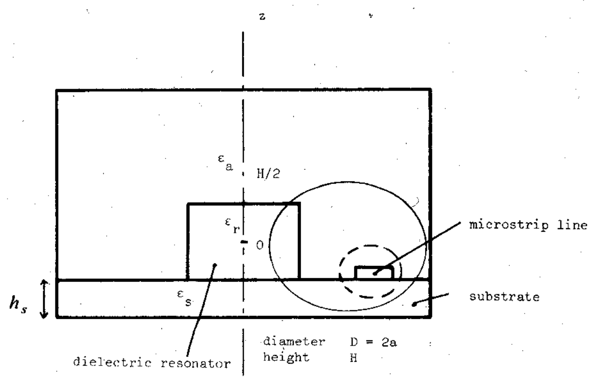

Fig. 1. Coupling between a microstripline and a dielectric resonator. —— magnetic line of the dielectric resonator, ---- line of the microstripline.

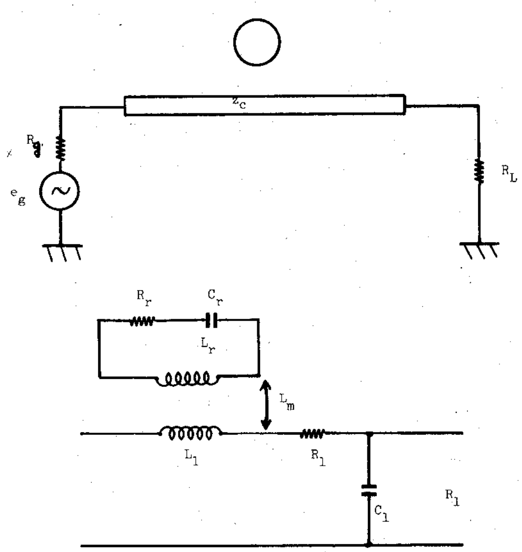

Fig. 2. Low-frequency equivalent network.

ω_0 and Q_0 are, respectively, the resonant pulsation and unloaded quality factor of the dielectric resonator inserted into the microstrip structure. The coupling coefficient characterized by the external quality factor Q_e is

$$Q_e = \frac{R_{\text{ext}}}{Z(\omega_0)} Q_0. \tag{3}$$

R_{ext} symbolizes the internal resistance of the generator (R_g) and the loaded impedance of the line (R_L) (Fig. 3). When the generator and the line are matched, i.e., $R_g = R_L = Z_c$, where Z_c is the characteristic impedance, the external quality factor expression is given by

$$Q_e = 2\frac{Z_c}{\omega_0}\left(\frac{L_m^2}{L_r}\right)^{-1}. \tag{4}$$

The external quality factor is a function of the distance separation between the line and the resonator by the factor L_m^2/L_r, which we will now evaluate.

The dielectric resonator acting on the $TE_{0\gamma\delta}$ mode can be assimilated to a magnetic dipole. Let I_r be the current flowing in the dielectric resonator. The voltage induced into the microstripline by this current is

$$e = j\omega L_m I_r. \tag{5}$$

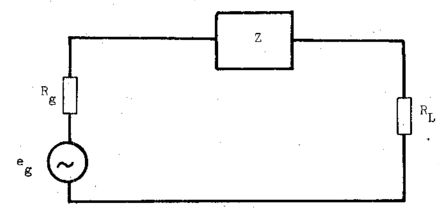

Fig. 3. Transformed equivalent network.

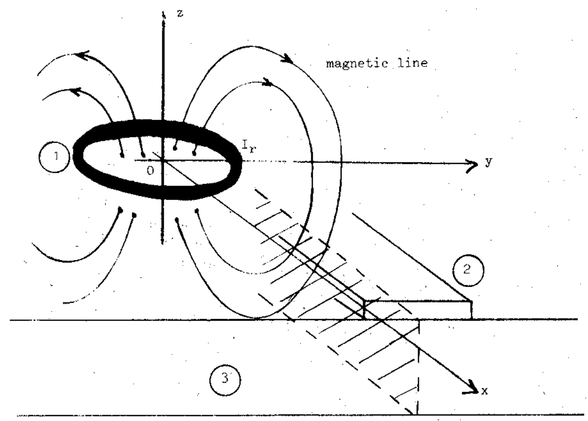

Fig. 4. $TE_{0\gamma\delta}$ dielectric resonator ≡ a magnetic dipole. ① Dielectric resonator-magnetic dipole. ② Microstripline. ③ Substrate.

This induced voltage can also be computed from the magnetic flux through the S section in the microstrip substrate (Fig. 4)

$$e = j\omega\mu_0 \int\int_S \vec{H}\cdot d\vec{S}. \tag{6}$$

The magnetic stored energy in the dielectric resonator is given by

$$\overline{W}_m = \tfrac{1}{4} L_r I_r^2. \tag{7}$$

Combining (5)–(7) and substituting into (4), the coupling is characterized by

$$Q_e = 4\frac{Z_c}{\omega_0 \mu_0^2} \frac{\overline{W}_m}{\left\{\int\int_S H\cdot dS\right\}^2} \tag{8}$$

$$\overline{W}_m = \frac{1}{4}\int \epsilon_0 \epsilon_i E_i E_i^* \, d\tau_i \tag{9}$$

where ϵ_i is the relative permittivity of the dielectric medium ($\epsilon_i = \epsilon_a, \epsilon_r, \epsilon_s$), and E_i is the electrical field component in the medium of permittivity ϵ_i.

B. Scattering Parameters

The dielectric resonator coupled with the microstripline is identical to a resonant parallel circuit placed in series with the line. It is a localized element for which we have established the lumped equivalent circuit shown in Fig. 5.

Let θ be the electrical length of the line; the parameters of the scattering matrix S of the system then satisfy [4]

$$S_{11} = \frac{Z(\omega_0)/Z_c}{2 + Z(\omega_0)/Z_c} e^{-2j\theta} \tag{10}$$

$$S_{12} = \frac{2}{2 + Z(\omega_0)/Z_c} e^{-2j\theta}. \tag{11}$$

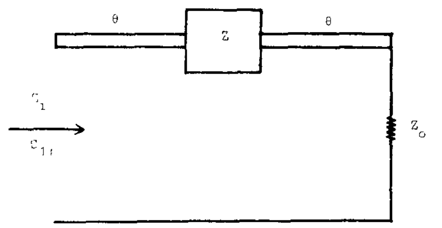

Fig. 5. Equivalent network (Z_t, S_{11}).

III. Computation

For the computation of the $Z(\omega_0)$, Q_e, and S_{ij} coefficients, it is necessary to evaluate the stored energy $\overline{W_m}$, the magnetic field $\vec{H}$ inside and outside the dielectric resonator, and also the integral $\int \vec{H}\cdot d\vec{S}$. Two methods are considered. The first uses the finite-difference method and the second an analytical one.

A. The Finite-Difference Method

A new method of computation for resonant frequencies and field plots of the revolution symmetrical modes of a cylindrical dielectric resonator has been previously introduced [10]. It also permits the evaluation of the stored energy $\overline{W_m}$.

For that we solve the Helmholtz equation expressed in cylindrical coordinates (12) by means of the finite-difference method

$$\frac{\partial^2 \Psi}{\partial r^2} + \frac{1}{r}\frac{\partial \Psi}{\partial r} + \frac{\partial^2 \Psi}{\partial z^2} + \left(k^2 + \frac{1}{r^2}\right)\Psi = 0 \qquad (12)$$

where $\Psi = E_\theta$ for the dipolar magnetic mode $TE_{0\gamma\delta}$.

Taking into account boundary conditions on metallic planes and continuity conditions at the dielectric interfaces, we obtain, using the finite-difference approximation [6], the linear system

$$A\psi = \lambda\psi$$

where A is the band matrix and λ is the eigenvalue function.

The resolution of this system permits the determination of the frequencies of the resonant system and the components of the electromagnetic field E_θ, H_r, H_z at any point of the composite dielectric structure.

Substituting the electric field components value E_θ into (9) and the magnetic field components into (8) to evaluate $\{\int HdS\}^2$, we obtain the external quality factor Q_e variations as a function of the distance between a straight line and the resonator (Fig. 6) and between a curved line and the resonator (Fig. 7).

B. The Analytical Method

The method used to compute the resonant frequency and EM field component is derived from the third approximation described in [7] and [8]. The walls of the dielectric resonator are supposed to be imperfect. The longitudinal magnetic field components in each zone of the structure

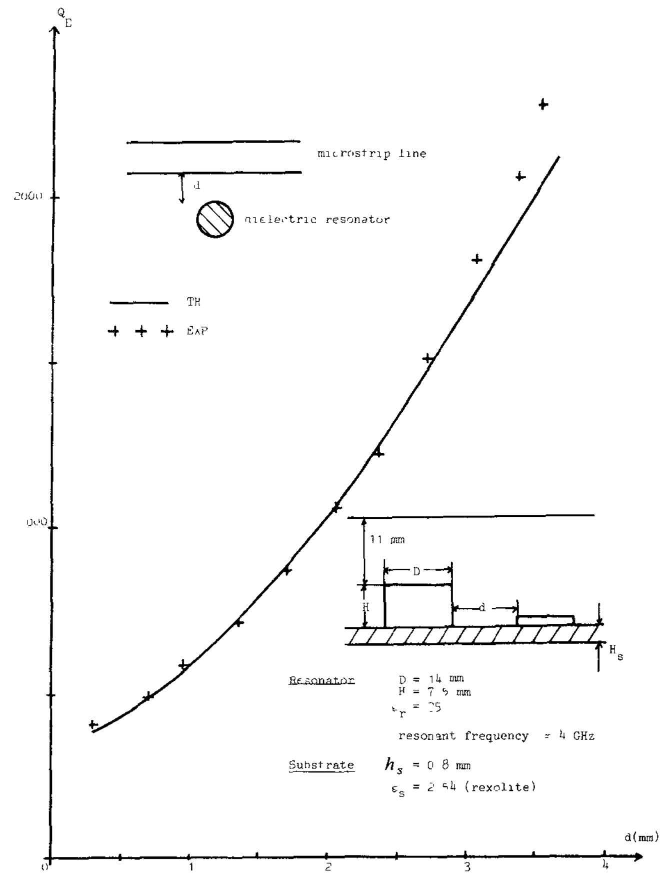

Fig. 6. $Q_e'S$ variations for a dielectric resonator coupled with a straight microstripline (numerical approach).

(Fig. 8) for the $TE_{0\gamma\delta}$ mode are given in (12)

$$H_{z_1} = H_0 J_0(k_r r)\{\cos\beta z + A_{21}\sin\beta z\}$$

$$H_{z_2} = 2H_0 J_0(k_r r) A_{31} e^{-\alpha_a d_3} sh\alpha_a(d_3 - z)$$

$$H_{z_3} = 2H_0 J_0(k_r r) A_{41} e^{-\alpha_s d_1} sh\alpha_s(d_1 + z)$$

$$H_{z_4} = H_0 A_{51}(\cos\beta z + A_{65}\sin\beta z) K_0(k_a r)$$

$$H_{z_5} = 2H_0 A_{81}(e^{-\alpha_a d_3} sh\alpha_a(d_3 - z) K_0(k_a r)$$

$$H_{z_6} = 2H_0 A_{71} e^{-\alpha_s d_1} sh\alpha_S(d_1 + z) K_0(k_a r)$$

where A_{ij} are constant values which are given in the Appendix, J_0 and K_0 are Bessel functions of the first and second order, respectively, and where

$$k_r^2 = \left(\frac{\omega}{c}\right)^2 \epsilon_r - \beta^2$$

$$\alpha_{s}^{a\,2} = k_r^2 - \left(\frac{\omega}{c}\right)^2 \epsilon_{s}^{a}$$

$$k_a^2 = \beta^2 - \left(\frac{\omega}{c}\right)^2 \epsilon_a.$$

The tangential components E_θ, H_r are deduced from Maxwell equations. They are, respectively, substituted into (9) and into $\int_s H\cdot dS$ to obtain the two expressions of these parameters, expressions which are given in the Appendix. So we can draw, using (8), the theoretical variations of Q_e

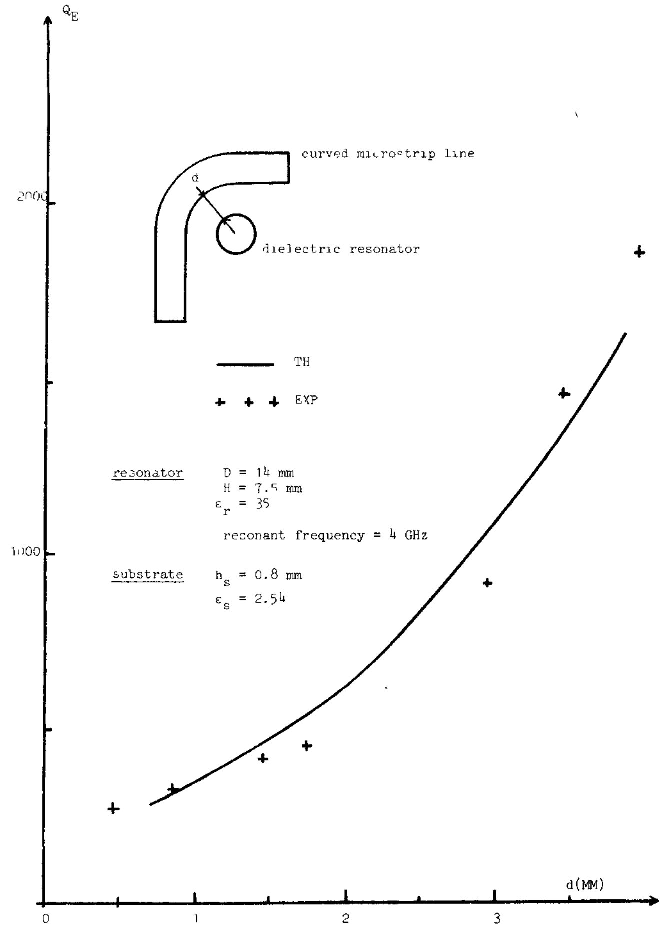

Fig. 7. $Q_e'S$ variations for a dielectric resonator coupled with a curved microstripline (numerical approach).

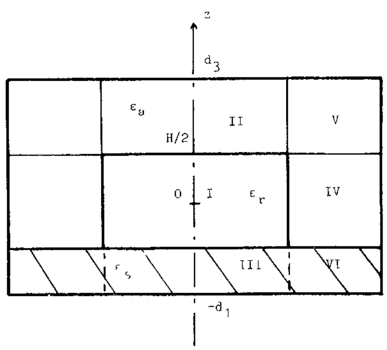

Fig. 8. Composite dielectric structure.

as a function of the distance between a straight line and a resonator (Fig. 9).

Note: In this analytical analysis, the resonant frequency of the structure is obtained by matching simultaneously tangential field components in $z = \pm H/2$ and $r = a$, respectively, and by solving simultaneously the two eigenvalue equations.

IV. Conclusion

Two methods have been presented to evaluate the external quality factor of a dielectric resonator coupled with a microstripline.

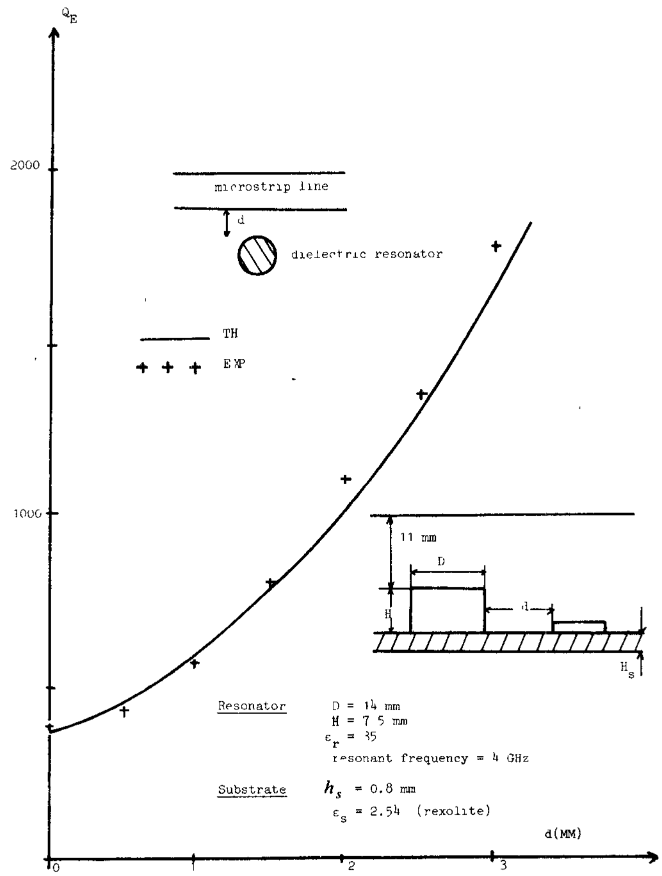

Fig. 9. $Q_e'S$ variations for a dielectric resonator coupled with a microstripline (analytical approach).

Good agreement between them and the experimental results has been observed.

The analytical method is easier to use than the finite-difference method. This latter justifies the utilization of the approximative analytical method.

Appendix

A_{ij} Expressions

$$A_{21} = A_{65} = \frac{\alpha_S - \beta th \alpha_S (d_1 - H/2) tg\beta H/2}{\alpha_S tg\beta \frac{H}{2} + \beta th \alpha_S (d_1 - H/2)}$$

$$A_{31} = \frac{\cos\beta H/2 + A_{21} \sin\beta H/2}{2e^{-\alpha_a d_3} sh\alpha_a (d_3 - H/2)}$$

$$A_{41} = \frac{\cos\beta H/2 - A_{21} \sin\beta H/2}{2e^{-\alpha_S d_1} sh\alpha_S (d_1 - H/2)}$$

$$A_{51} = \frac{J_0(k_r a)}{K_0(k_a a)}$$

$$A_{71} = A_{41} \cdot A_{51}$$

$$A_{81} = A_{41} \cdot A_{31}.$$

Stored Energy

$$W = H_0^2 \frac{\omega^2 \mu_0^2 \epsilon_0 \pi}{2} \Bigg\{ \frac{a^2}{k_r^2} \Bigg\{ J_1^2(k_r a) + J_0^2(k_r a)$$

$$- \frac{2 J_0(k_r a) J_1(k_r a)}{k_r a} \Bigg\}$$

$$\times \Bigg\{ \frac{H\epsilon_r}{2} \Bigg\{ 1 + \frac{\sin\beta H}{\beta H} + A_{21}^2 \Bigg(1 - \frac{\sin\beta H}{\beta H} \Bigg) \Bigg\}$$

$$+ \frac{A_{31}^2}{\alpha_a} e^{-2\alpha_a d_3} \{ sh 2\alpha_a (d_3 - H/2)$$

$$- 2\alpha_a (d_3 - H/2) + \frac{A_{41}^2}{\alpha_S} e^{2\alpha_S d_1} \{ sh 2\alpha_S (d_1 - H/2)$$

$$- 2\alpha_S (d_1 - H/2) + \frac{A_{51}^2 H}{2 k_A^2} \Bigg\{ b^2 \Bigg\{ K_1^2(k_a b) - K_0^2(k_a b)$$

$$- 2 \frac{K_0(k_a b) K_1(k_a b)}{k_a b} \Bigg\} - a^2 \Bigg\{ K_1^2(k_a a) - K_0^2(k_a a)$$

$$- 2 \frac{K_0(k_a a) K_1(k_a a)}{k_a a} \Bigg\}$$

$$\cdot \Bigg\{ 1 + \frac{\sin\beta H}{\beta H} + A_{21}^2 \Bigg(1 - \frac{\sin\beta H}{\beta H} \Bigg) \Bigg\} \Bigg\}$$

Integral Value

$$\left\{ \int \vec{H} \cdot \vec{dS} \right\}^2 = 4 H_0^2 A_{71}^2 \frac{\pi}{k_a^4} e^{-2\alpha_S d_1} e^{-2 k_a d} sh \alpha_S (d_1 - H/2)$$

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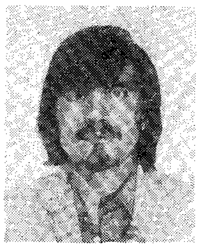

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